2024 DCAA Problems

October 3, 2023

Note: Most projects have a programming component. Many math students report not liking programming, but the reality is that research mathematics often involves (or at least can benefit from) some level of computer programming. In addition, many non-academic jobs have a programming component, and we want to help you develop a skill that will give you greater versatility in the job market. Since many mathematics students do not have much programming experience, we will be using time during the first week of the REU to teach you some basic programming skills in Python, widely considered one of the easiest programming languages to learn and also frequently used by mathematicians.

There are three main points to consider when indicating your interest in a problem in the application. The first is that if you don't have much programming experience, that is OK–you should still apply if you are interested; when a project description below states that students will use programming to do something, that level of programming should be covered by the training mentioned above. The second is that when the *prerequisites* suggest that programming experience would be a plus, it would be best if you have had (or will have had) one or two semesters of programming under your belt when you arrive at the REU. Finally, if you are motivated to pursue a project, we will take that into consideration when thinking about your programming experience.

Also, your interest in a problem at this point is just to help us find students for each mentor. You will be able to change your mind after you arrive and learn more about the problems.

Possible Research Projects

Random Triangles on Lattices (a.k.a., "The Triangle Problem")

Mentor: Dr. Robert Niichel

Lewis Carroll presented in the his book *Pillow Problems* [3] the following text: "Choosing three random points on the infinite plane, what is the probability that the triangle formed is obtuse?". The problem is not well-defined (there is no uniform probability distribution on the infinite plane), but it is nevertheless an interesting problem for consideration. Because the problem is ill-defined, there are a number of ways to view the situation (for example, [5]), and in fact a few different numerical answers have been proposed.

One way to tackle the problem is to restrict attention to a finite region in the plane–for instance, what is the probability of producing an obtuse triangle if the points are selected at random from the unit square? This problem *is* well-defined, and the solution has been found definitively. Moreover, different probability distributions can be applied to the unit square which can yield different and interesting results.

We will be working on will be a discrete version of this problem.

Prerequisites: Basic probability theory and counting techniques, linear algebra and/or multivariable calculus

Control Theory: The Kalman Filter

Mentor: Dr. Nick Wintz

Control theory is a branch of mathematics that in which we steer some process or observe some or all of its components. Here, the information of our process is determined by a dynamic equation. Traditionally, applications have concepts in electrical and mechanical engineering, finance, and exercise science. However, in recent years, there has been an interest in control theory for more health-related purposes, from tracking disease/vaccination status to offering drug rehabilitation strategies. In particular, one of the fundamental tools of controls is the Kalman filter.

Sometimes a process cannot be accurately described due to corrupted or missing measurements. The Kalman filter is an algorithm that seeks to minimize the difference between the true process and our estimate. Initially, there is a "time update" in which the algorithm predicts the process estimate based on a previous measurement. This prediction is then associated with an error covariance. Next there is a "measurement update" in which the algorithm calculates a correction of the process estimate based on the prediction and the new measurement along with its associated error covariance. The algorithm then repeats itself. The filter is widely used for two reasons. The first, the filter is very effective in a wide of range of applications, including navigation, ballistics, image processing, among several others. The other is its relative ease to implement and modify.

In recent years, there has also been merging concepts of control within those in data science. In this project, we implement the Kalman filter on an available data set.

Prerequisites: Linear Algebra, Differential Equations

Data Science Project: Classroom Usage

Mentors: Dr. Robert Niichel, Mr. Tyler Davis

Project Consultant: Dr. Heath Howard, Director of Institutional Research, Fairmont State University

Students interested in learning about data science or in honing their skills on real-world problems are invited to indicate their interest in this problem. Each semester, Fairmont State University must generate an academic schedule which accommodates thousands of students enrolled in hundreds of classes across every academic program. In order to generate an effective course schedule, university leadership must synchronize the capability of facilities, the academic needs of students, and the bandwidth of university instructors. To accomplish this efficiently, the university needs to leverage data science. Students working on this project will be tasked with finding optimal ways to assign classrooms in a system with hundreds of constraints and variables.

Project students will need to consult regularly with Fairmont State's Office of Institutional Research to determine whether the solution(s) they are proposing meet the actual needs of the university. In addition to the faculty mentor, a Fairmont State alumnus, formerly a Senior Data Analyst for JPMorgan Chase and CVS Health, and currently a Senior Data Analyst for iRobot, will also be working directly with students."

Prerequisites: Probability and/or Statistics. Some programming experience would be beneficial.

The Nonnegative Inverse Eigenvalue Problem

Mentor: Dr. Bishnu Sedai

The inverse eigenvalue problem concerns the construction of a matrix from prescribed spectral data and arise in many applications, including control design, seismic tomography, geophysics, molecular spectroscopy, particle physics, structural analysis, circuit theory, and mechanical system simulation. The nonnegative inverse eigenvalue problem (NIEP) is concerned with determining the necessary and sufficient conditions for a given list of n com- plex numbers $\sigma = (\lambda_1, \lambda_2, \ldots \lambda_n)$ to be the spectrum of an $n \times n$ nonnegative matrix A. If there exists an $n \times n$ nonnegative matrix A with spectrum σ, we will say that σ is realizable and that A realizes σ. The NIEP has been an open question in matrix theory and regarded as one of the most difficult in linear algebra and matrix theory over the past several years. The problem is completely solved for $n \leq 4$ [4] and for $n = 5$ only when the trace of A is zero [8].

In this project, students will work on making progress on solutions of the NIEP for $n \geq 6$. Students will work on a constructive method and use MATLAB or Python to discover the class of matrices that realize σ.

Prerequisites: Linear Algebra

Contrast in EEG hemispheric balance while listening to nature sounds versus artificial sounds

Mentors: Dr. Mahmood Hossain, Dr. Robert Niichel

Project Consultant: Dr. Tadashi Kato

In recent years, there has been increasing scientific attention given to the hemispheric synchronicity of neurophysiological activities during altered states of consciousness, including but not limited to, state of meditation [7] or exposure to relaxing music [2]. In this proposed study, REU students will be invited to investigate the differential effects between nature sounds versus artificial sounds over EEG hemispheric balance.

Students who participate in this project will be provided with a data-set of electroencephalogram (EEG) of approximately 40 healthy adults while listening to three 1-minute-long audio-tracks of nature sounds (such as ocean waves, waterfall, and rain) as well as three 1-minute-long audio-tracks of artificial sounds (such as sprinkler, helicopter, and boat). Students will be encouraged to be engaged in hemispheric coherency analyses by applying several different data science methods.

Prerequisites: Statistics or probability. Some programming experience would be helpful.

Singmaster's Conjecture

Mentor: Dr. James Long

One well-known structure in combinatorial mathematics is *Pascal's Triangle*. It is not hard to see that no number n can appear after the n-th row (besides 1, which appears on each row). In 1971 British mathematician Singmaster proposed the following conjecture.

Conjecture 1 *If $N(j)$ is the number of times j appears in Pascal's Triangle, then there exists some constant C such that $N(j) < C$ for all integers $j > 1$. In big O notation, $N(j) = O(1)$*

Singmaster showed $N(j) = O(\log(j))$ [9]. Abbott, Erdős, and Hansen improved this to $O\left(\frac{\log(j)}{\log(\log(j))}\right)$ [1]. Kane improved the bound in 2004, then again in 2007 [6], the latter bound being $N(j) = O\left(\frac{\log(j)\log(\log(\log(j)))}{\log(\log(j))^3}\right)$ and the former having one less $\log(\log(j))$ factor.

Student researchers will design and implement a reasonably efficient Python program to calculate the first several rows Pascal's Triangle and track appearances of entries therein. We will then explore Singmaster's conjecture and some potential sub-problems.

Prerequisites: Basic Probability and combinatorics

References

[1] H. L. Abbott, P. Erdos, and D. Hanson. On the number of times an integer occurs as a binomial coefficient. *The American Mathematical Monthly*, 81(3):256–261, 1974.

[2] E. Baeck. The neural networks of music. *European Journal of Neurology*, 9(5):449–456, 2002.

[3] Lewis Carroll. *Curiosa Mathematica. Part II. Pillow-Problems Thought Out During Wakeful Hours. By Charles L. Dodgson.* Macmillan and Company, 1893.

[4] Moody T. Chu and Gene H. Golub. Structured inverse eigenvalue problems. *Acta Numerica*, 11:1–71, 2002.

[5] Clayton Shonkwiler Jason Cantarella, Tom Needham and Gavin Stewart. Random triangles and polygons in the plane. *The American Mathematical Monthly*, 126(2):113–134, 2019.

[6] Daniel M. Kane. Improved bounds on the number of ways of expressing as a binomial coefficient. *Integers*, 7, 2007.

[7] Tadashi Kato and Kimiko Kawano. Physiological investigation on eeg coherency during yoga meditation and qigong exercise. *Journal of International Society of Life Information Science*, 35(2):99, 2017.

[8] Thomas J. Laffey and Eleanor Meehan. A characterization of trace zero nonnegative 5×5 matrices. *Linear Algebra and its Applications*, 302-303:295–302, 1999.

[9] David Singmaster. How often does an integer occur as a binomial coefficient? *The American Mathematical Monthly*, 78(4):385–386, 1971.